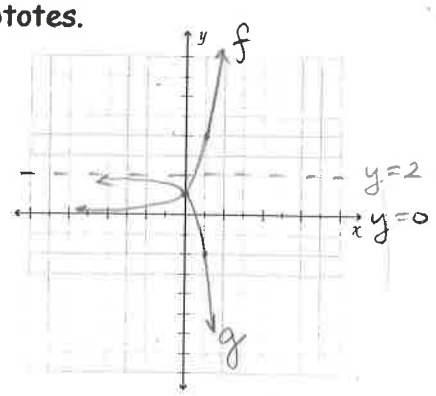
PC 4 and SL 7 Review

Name Key

1. Graph f(x) and g(x) on the same graph. Label asymptotes.

$f(x) = 4^x$

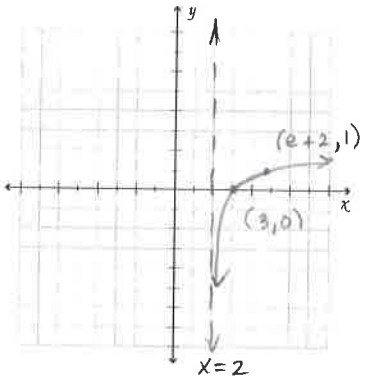
$g(x) = -4^x + 2$



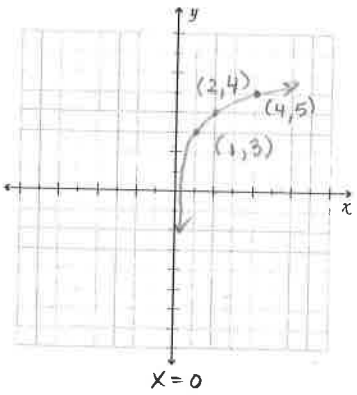
Graph. Label asymptotes with their equations. State domain and range.

2. $y = \ln(x-2)$

Domain: $(2, \infty)$
Range: $\mathbb{R}$



3. $y = 3 + \log_2 x$



4. If you have $10,000 and want to put it in a savings account, would you make more in an account that pays 4.5% interest monthly for 10 years or in an account that pays 5% interest quarterly for 10 years?

$A = 10{,}000\left(1 + \frac{0.045}{12}\right)^{12(10)}$

$= 10{,}000\left(\frac{12.045}{12}\right)^{120}$

$\approx \$15{,}669.93$

$A = 10{,}000\left(1 + \frac{0.05}{4}\right)^{4(10)}$

$= 10{,}000\left(\frac{4.05}{4}\right)^{40}$

$\approx \$16{,}436.19$

You would make $766.26 more investing @ 5% compounded quarterly.

b) How much would you make in an account compounded continuously at 4%?

$A = 10{,}000e^{.04(10)}$

$= 10{,}000e^{.4} \approx \$14{,}918.25$

$14{,}918.25 - 10{,}000.00$

You would make $4,918.25 on the investment.

c) How long would it take to double your money if you invested at 8.2% compounded continuously?

$20{,}000 = 10{,}000e^{.082t}$

$2 = e^{.082t}$

$\ln 2 = .082t \rightarrow t = \frac{\ln 2}{.082}$

$t \approx 8.45$ years.

d) What is the effective annual yield at 6.8% compounded quarterly?

$1 + r = \left(1 + \frac{.068}{4}\right)^4$

$1 + r = \left(\frac{4.068}{4}\right)^4 \rightarrow r = (1.017)^4 - 1$

$r \approx .0698$ or 6.98%

5. Evaluate the expressions without using a calculator:

a) $\log_2(1/8)$

$\log_2 2^{-3}$

-3

b) $\log_3\sqrt{3}$

$\log_3 3^{1/2}$

$\frac{1}{2}$

c) $\log 0.01$

$\log_{10} 10^{-2}$

-2

d) $\log_2 3.2 + \log_2 5$

$\log_2(3.2 \cdot 5)$

$\log_2 16$

$\log_2 2^4$

4

6. Solve for x.

a) $\log_x(1/3) = 3$

$(x^3)^{1/3} = \left(\frac{1}{3}\right)^{1/3}$

$x = \sqrt[3]{\frac{1}{3}} \cdot \frac{\sqrt[3]{3^2}}{\sqrt[3]{3^2}}$

$x = \frac{\sqrt[3]{9}}{3} \approx 0.69$

b) $\log_3 81 = x + 1$

$3^{x+1} = 81$

$3^{x+1} = 3^4$

$x + 1 = 4$

$x = 3$

c) $\log(x+3) - \log x = \log x + \log 2$

$\log\left(\frac{x+3}{x}\right) = \log 2x$

$x \cdot \frac{x+3}{x} = 2x \cdot x$

$x + 3 = 2x^2$

$0 = 2x^2 - x - 3$

$0 = (2x-3)(x+1)$

$x = \frac{3}{2}, -1$ ~~-1~~ can't take log of neg.

d) $2\ln x^3 = 18$

$6\ln x = 18$

$\ln x = 3$

$x = e^3$

e) $\frac{2}{1-e^x} = 3(1-e^x)$

$\frac{2}{3} = 1 - e^x$

$e^x = \frac{1}{3}$

$\ln\frac{1}{3} = x$

$x \approx -1.0986$

f) $e^{2x} + 6e^x - 16 = 0$

$(e^x + 8)(e^x - 2) = 0$

~~$e^x = -8$~~ $e^x = 2$

$\ln 2 = x$

$x \approx 0.6931$

g) $3^{2x} - 3^{x+2} + 8 = 0$

$(3^x)^2 - 3^2 \cdot 3^x + 8 = 0$

$(3^x - 1)(3^x - 8) = 0$

$3^x = 1$ $\quad 3^x = 8$

$x = 0$ $\quad x\frac{\ln 3}{\ln 3} = \frac{\ln 8}{\ln 3}$

$x \approx 1.8928$

h) $5^{2x} = \frac{1}{125}$

$5^{2x} = 5^{-3}$

$2x = -3$

$x = -\frac{3}{2}$

i) $7^{2x-6} = 1$

$2x - 6 = 0$

$2x = 6$

$x = 3$

j) $(\log x)^2 - 6\log x = -7$

Let $y = \log x$

$y^2 - 6y + 9 = -7 + 9$

$(y-3)^2 = 2$

$y = 3 \pm \sqrt{2}$

$\log x = 3 \pm \sqrt{2}$

$x = 10^{(3\pm\sqrt{2})} \approx \begin{cases} 25{,}954.55 \\ 38.53 \end{cases}$

k) $\log_x(3x^2 + 10x) = 3$

$x^3 = 3x^2 + 10x$

$x(x^2 - 3x - 10) = 0$

$x(x-5)(x+2) = 0$

$x = \cancel{0}, 5, \cancel{-2}$ extraneous

def. of $\log_a x$

$a > 0$ and $\neq 1$

l) $\log(x^2+1) - 2\log x = 1$

$\log(x^2+1) - \log x^2 = 1$

$\log\left(\frac{x^2+1}{x^2}\right) = 1$

$10^1 = \left(\frac{x^2+1}{x^2}\right)$

$10x^2 = x^2 + 1$

$9x^2 = 1$

$x = \pm\sqrt{\frac{1}{9}} = \frac{1}{3}$

extraneous

7. Solve for x giving your answer to 4 decimal places:

a) $\log_x e = 1/2$

$(x^{1/2})^2 = (e)^2$

$x = e^2$

$x \approx 7.3891$

b) $\log_e(x+2) = 4$

$e^4 = x + 2$

$x = e^4 - 2$

$x \approx 52.5982$

c) $\log_x e = -2$

$(x^{-2})^{-1/2} = (e)^{-1/2}$

$x = e^{-1/2}$

$x \approx 0.6065$

d) $10^{-2x} = 2$

$\log 2 = -2x$

$x = \frac{\log 2}{-2}$

$x \approx -0.1505$

e) $3^{4x+1} = 10$

$\log 3^{4x+1} = \log 10$

$(4x+1)\log 3 = 1$

$4x\log 3 + \log 3 = 1$

$\frac{4x\log 3}{4\log 3} = \frac{1 - \log 3}{4\log 3}$

$x \approx 0.2740$

f) $\frac{3^x}{3^x + 3} = 1/3\,(3^x + 3)$

$3^x = \frac{1}{3} \cdot 3^x + 1$

$3^x - \frac{1}{3} \cdot 3^x = 1$

$\frac{2}{3} \cdot 3^x = 1$

$3^x = \frac{3}{2}$

$\frac{x\ln 3}{\ln 3} = \frac{\ln\left(\frac{3}{2}\right)}{\ln 3}$

$x \approx 0.3691$